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FIRST SEMESTER | **B.Tech.(ALL)**

END SEMESTER EXAMINATION | **NOVEMBER 2024**

AM101/MA101 (MATHEMATICS I)

Time: 3 hours | **Maximum Marks: 50**

Note: Attempt any five questions.
All questions carry equal marks.
Assume suitable missing data, if any.

Q1(i) Given $u = e^{rcos\theta}\cos(rsin\theta)$, $v = e^{rcos\theta}\sin(rsin\theta)$, Prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$ (CO3).

(ii) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show by Euler's theorem that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\,cotu$$ (CO3)

Q2(i) A rectangular box which is open at the top, has a capacity of 256 cubic feet. Determine the dimensions of the box such that the least material is required for the construction of the box. (CO3)

(ii) Use Gauss Divergence theorem to evaluate

$$\iint_S [(x^3 - yz)i - 2x^2y\,j + 2k].\hat{n}\,ds$$

Where S denotes the surface of the cube bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a.$ (CO5)

Q3(i) Evaluate the double integral $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration. (CO4)

(ii) Find by double integration, the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$. (CO4)

Q4(i) Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{(a^2-r^2)/a} r\,dr\,d\theta\,dz$ (CO4)

(ii) Find the directional derivative of $\nabla.(\nabla\emptyset)$ at the point (1,-2,1) in the direction of the normal to the surface

$xy^2z = 3x + z^2, where\ \emptyset = 2x^3y^2z^4.$ (CO5)

Q5(i) A vector field is given by $\vec{F} = (x^2 - y^2 + x)i - (2xy + y)j$. Show that the field is irrotational and find its scalar potential.

$\Delta \times \vec{F}$

(CO5)

(ii) Verify Stoke's theorem for the vector point function $\vec{F} = (x + y)i + (2x - z)j + (y + z)k$ over the surface of a triangular lamina with vertices (2,0,0), (0,3,0) and (0,0,6).

(CO5)

Q6(i) Test the convergence of the series

$1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \text{-------}$ (CO1)

(ii) Find the perimeter of the loop of the curve

$3ay^2 = x^2(a - x)$ (CO2)