Total No. of Page: 02 Roll No. ....

FIRST SEMESTER B. Tech.

END SEM EXAMINATION NOVEMBER-2018

MA – 101 Mathematics I

Time: 3:00 Hours *Max. Marks: 50*

Note: Answer any five questions. All questions carry equal marks.

Assume suitable missing data, if any.

1.

a) Test the convergence of the following series

$$\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\cdots$$

b) Expand $\sin x$ in powers of $(x-\frac{\pi}{2})$ and determine the value of $\sin 91^0$ correct upto four decimal places.

2.

a) Examine the function $sinx+\sin y+\sin(x+y)$ for extreme points and also find the value of the function at those points.

b) If $u = cosec^{-1}\left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}(13+\tan^2 u)$.

3.

a) Change the order of integration in the integral and hence

$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dx\, dy$ and hence evaluate.

b) Evaluate $I=\iiint \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\, dx\, dy\, dz$ over the region

$V=\left\{(x,y,z); x\geq 0, y\geq 0,\ z\geq 0, \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq 1\right\}$.

4.

a) Find the area lying inside the curve $r = a\,sin\theta$ and outside the curve $r = a\,(1 - \cos\theta)$.

b) Find the area of the surface formed by the revolution of $y^2 = 4ax$ about the x- axis, by an arc from the vertex to one end of the latus rectum.

5.

a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). In what direction it will be maximum? Find also the magnitude of this maximum.

b) Verify Green's theorem in the plane for $\oint_C \{(3x^2 - 8y^2)dx + (4y - 6xy)dy\}$ where C is the boundary of the region defined by $y = x \; and \; y = x^2$.

6.

a) Find the total work done in moving a particle in a force field $\vec{f} = 3xy\,\hat{i} - 5z\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

b) Prove that Curl(Curl $\vec{V}$) $= grad\ div\ \vec{V} - \nabla^2\vec{V}$

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